REFERENCES

1. SUQUET P., Analyse limite et homogénéisation, C.R. Acad. Sci. Paris, Ser. II, Mec., 296, 18, 1983.
2. DEMENGEL F. and TANG Qi, Homogénéisation en plasticité, C.R., Acad. Sci . Paris, Ser. I, 303, 8, 1986.
3. ZHIKOV V.V., Averaging of functionals of calculus of variations and elasticity theory, Izv. Akad. Nauk. Ser. Mat., 50, 4, 1986.
4. SEREGIN G.A., Expansion of the variational formulation of the problem for a rigidly plastic medium to a velocity field with slip-type discontinuities, PMM, 47, 6, 1983.
5. KAMENYARZH YA.A., Conditions on surfaces of discontinuity in a rigidly plastic analysis, Dokl. Akad. Nauk SSSR, 286, 3, 1986.
6. BERDICHEVSKII V.L., Variational Principles of the Mechanics of a Continuous Medium. Nauka, Moscow, 1983.
7. KAMENYARZH YA.A., On formulations of problems of the theory of ideal plasticity, PMM, 47, 3, 1983.

Translated by M.D.F.

PMM U.S.S.R.,Vol.53,No.5,pp.654-659,1989
Printed in Great Britain

0021-8928/89 $10.00+0.00

A CRACK ON THE INTERFACIAL BOUNDARY OF PRESTRESSED ELASTIC MEDIA*

V.B. ZELENTSOV and L.M. FILIPPOVA

The plane problem of the equilibrium of a piecewise-homogeneous body weakened by a crack located on the interfacial boundary of the materials and under uniform loading is considered. There are initial stresses in the body that act in the direction of the interfacial boundary. The solution of the problem is found by reduction to a system of singular integral equations. It is established that exactly as in an analogous problem without taking account of the initial stresses /1-3/, the solution near the crack tip is rapidly oscillating in nature, where the oscillation zone is broadened as the initial compression increases.

1. We consider a piecewise-homogeneous elastic body consisting of two half-planes interconnected along the whole interfacial boundary $y=0$ with the exception of the segment $|x|<1$ which is a rectilinear crack in the form of an infinitely thin slit. Here x, y are dimensionless coordinates referred to the crack length a. The body is subjected to a preliminary homogeneous finite strain for which there are no stresses on lines parallel to the x axis. The crack edges are loaded by uniform pressure p and a uniform shearing load of intensity τ. The strain caused by the loading of the crack edges is assumed to be small, and consequently, we use linearized equilibrium equations for a prestressed medium to solve the problem /4/.

For non-linearly elastic materials of general form the solution of the problem gives rise to serious technical difficulties. Consequently, we will investigate specific models of materials. It is assumed in this section that the materials filling the lower and upper half-planes are incompressible and described by the Mooney model /4, 5/ with shear modulus G_1 in the lower $y<0$ half-plane and shear modulus G_2 in the upper $y<0$ half-plane.

The mathematical formulation of the problem constains boundary conditions on the line $y=0$

$$\begin{gathered}u_1=u_2,\quad v_1=v_2,\quad \theta_{yy1}=\theta_{yy2},\quad \theta_{yx2}=\theta_{yx1},\quad 1<|x|<\infty\\ \theta_{yy1}=\theta_{yy2}=-p,\quad \theta_{yx1}=\theta_{yx2}=\tau,\quad |x|\leqslant 1\end{gathered}\tag{1.1}$$

**Prikl.Matem.Mekhan.*,53,5,830-836,1989

in addition to the differential equations presented in /4/.

Here u, v are the horizontal and vertical components on the displacement, and θ_{yx}, θ_{yy} are the Piola linearized stress tensor components. The subscript 1 refers to the lower and the subscript 2 to the upper half-plane.

By using the Fourier integral transformation the problem formulated in the class of functions decreasing at infinity can be reduced to a system of singular integral equations in the unknown displacement jumps at the slit $y = 0$, $|x| < 1$

$$\frac{\delta}{\Delta^2} u'(x) - \frac{1}{\pi} \int_{-1}^{1} \frac{v(\xi)}{(\xi - x)^2} d\xi = p_*, \quad |x| \leqslant 1 \tag{1.2}$$

$$\delta v'(x) + \frac{1}{\pi} \int_{-1}^{1} \frac{u(\xi)}{(\xi - x)^2} d\xi = \tau_*, \quad |x| \leqslant 1$$

$$u = u_1|_{y=0} - u_2|_{y=0}, \quad v = v_1|_{y=0} - v_2|_{y=0}$$

$$p_*' = pR\xi(\Delta, \varepsilon) a, \quad \tau_* = \tau R \Delta^2 \xi(\Delta, \varepsilon) a$$

$$\xi(\Delta, \varepsilon) = L(\Delta)(1 - \varepsilon^2) - 2(1 + \varepsilon)\Delta^2(\Delta^2 + 1)^2 + 2(1 - \varepsilon)\cdot (\Delta^2 - 1)^2$$

$$L(\Delta) = \Delta^6 + \Delta^4 + 3\Delta^2 - 1, \quad R^{-1} = G_1 \varepsilon (1 + \varepsilon)(1 + \Delta^2)$$

$$\delta = \frac{1 - \varepsilon}{1 + \varepsilon} \frac{\Delta^2 - 1}{\Delta^2 + 1}, \quad \varepsilon = \frac{G_1}{G_2}$$

Here $\Delta = 1 + e > 0$, where e is the magnitude of the preliminary homogeneous strain along the x-axis and the prime denotes the derivative with respect to x.

The system of integral equations obtained reduces to the Hilbert problem on the slit. Using /6/, we obtain the solution of the system in the form ($|x| \leqslant 1$)

$$\begin{gathered} u(x) = -\Delta \chi(\Delta, \varepsilon)[\tau\Delta \cos \omega(x) + p \sin \omega(x)](1 - x^2)^{1/2} \\ v(x) = \chi(\Delta, \varepsilon)[p \cos \omega(x) - \tau\Delta \sin \omega(x)](1 - x^2)^{1/2} \\ \chi(\Delta, \varepsilon) = aR^{-1}\xi(\Delta, \varepsilon) L^{-1}(\Delta), \quad \omega(x) = \gamma \ln(1 + x)/(1 - x) \\ \gamma = (2\pi)^{-1} \ln \alpha, \quad \alpha = (\Delta - \delta)/(\Delta + \delta) \end{gathered} \tag{1.3}$$

The solutions (1.3) found determine the divergence of the crack edges and enable the displacement field to be obtained in the body. In particular, the displacements in the lower half-plane ($y \leqslant 0$) are calculated from the formulas

$$\begin{gathered} v_1(x, y) = aK_1(\Delta, \varepsilon)\{-\Delta l_{11} \operatorname{Re} F_1(z_1) - l_{12} \operatorname{Im} F_2(z_1) + \Delta l_{21} \operatorname{Re} F_1(z) + l_{22} \operatorname{Im} F_2(z)\} \\ F_1(z) = (z^2 - 1)^{1/2}[p \sin \omega_* - \tau\Delta \cos \omega_*] + \Delta\tau z \\ F_2(z) = -(z^2 - 1)^{1/2}[p \sin \omega_* + \tau\Delta \sin \omega_*] + pz \\ l_{11} = -\varepsilon(\Delta^2 - 1) - 4\varepsilon\Delta^2(1 - \varepsilon) + \varepsilon(1 + \Delta^4)[2 - \varepsilon(1 + \Delta^4)] \\ l_{12} = -2\varepsilon(\Delta^4 - 1)\Delta^2 - 2\varepsilon\Delta^2(1 + \Delta^4 - 2\varepsilon) + \varepsilon(1 - \varepsilon)(1 + \Delta^4)^2 \\ l_{21} = \varepsilon(1 + \Delta^4)(\Delta^2 - 1)^2 + 4\varepsilon(1 + \varepsilon)\Delta^2 - \varepsilon(1 + \Delta^4)[2\Delta^2 + \varepsilon(1 + \Delta^4)], \\ l_{22} = 2\varepsilon(\Delta^2 - 1)^2\Delta^2 + 2\varepsilon\Delta^2(1 + \Delta^4 + 2\varepsilon\Delta^2) - \varepsilon(1 + \varepsilon)\Delta^2(1 + \Delta^4)^2 \\ K_1^{-1}(\Delta, \varepsilon) = G_1 R(\Delta^2 - 1) L(\Delta), \quad \omega_* = \gamma \ln(z + 1)/(z - 1) \\ z_1 = x + iy\Delta^2, \quad z = x + iy \end{gathered} \tag{1.4}$$

$$\begin{gathered} u_1(x, y) = aK_1(\Delta, \varepsilon)\{\Delta^2 l_{11} \operatorname{Im} F_3(z_1) - \Delta^2 l_{12} \operatorname{Re} F_{11}(z_1) - \Delta l_{11} \operatorname{Im} F_3(z) + l_{22} \operatorname{Re} F_4(z)\} \\ F_3(z) = (z^2 - 1)^{1/2}(p \sin \omega(z) + \tau\Delta \cos \omega(z)) + \tau\Delta z \\ F_4(z) = (z^2 - 1)^{1/2}(p \cos \omega(z) - \tau\Delta \sin \omega(z)) - pz \end{gathered} \tag{1.5}$$

An expression for the displacement of the lower crack edge results from (1.4) and (1.5)

$$\begin{gathered} v_1(x) = 0{,}5a\varepsilon R^{-1}L^{-1}(\Delta)[(pm_- \cos \omega + \tau\Delta m_+ \sin \omega)(1 - x^2)^{1/2} - 4\Delta^3(1 - \Delta^4)x\tau], \quad |x| \leqslant 1 \\ m_\pm = 4\Delta(1 - \Delta^4) \operatorname{sh} \pi\gamma \pm 2g(\Delta, \varepsilon) \\ g(\Delta, \varepsilon) = \Delta^6 + 3\Delta^4 - \Delta^2 + 1 + \varepsilon L(\Delta) \end{gathered} \tag{1.6}$$

$$v_1(x) = -2\varepsilon a\Delta(1 - \Delta^4) R^{-1}L^{-1}(\Delta)[(p \sin \omega + \Delta\tau \cos \omega)(x^2 - 1)^{1/2} + \tau\Delta x], \quad |x| > 1 \tag{1.7}$$

$$u_1(x) = -a\varepsilon R^{-1}L^{-1}(\Delta)\{(pn_- \sin \omega - \tau\Delta n_+ \cos \omega)(1 - x^2)^{1/2} - 2\Delta^2(\Delta^4 - 1)x\}, \quad |x| \leqslant 1 \tag{1.8}$$

$$n_{\mp} = 2\Delta^2 (\Delta^4 - 1) \operatorname{sh} \pi\gamma \mp \Delta g (\Delta, \varepsilon) \tag{1.9}$$
$$u_1 (x) = -2a\varepsilon (\Delta^4 - 1) R^{-1}L^{-1} (\Delta) [(p \cos \omega - \tau\Delta \sin \omega) (x^2 - 1)^{1/2} - px], \quad | x | > 1$$

Components characterizing the body displacement as an absolutely rigid solid are discarded in (1.4)-(1.9).

As in /1-3/, the relative displacements u, v of the crack edges oscillate on approaching the crack apex, where the oscillation frequency tends to infinity as $| r | \to 1$ Analysis of the magnitude of the oscillation zone enables the interval of physical correctness of the problems being solved to be indicated.

Three cases can be represented for Mooney materials.

For $\varepsilon \neq 1$ (different materials) and for $\Delta = 1$ (no prestresses) we have $\gamma = 0$, i.e., no oscillations. The system of integral equations decomposes here into two individual equations. This essential difference from the analogous problem for an unstressed medium /1-3/ is due to incompressibility of the material.

For $\varepsilon = 1$ (identical materials) and $\Delta \neq 1$ (prestresses present) there are also no oscillations. Note that in this case the crack edges experience vertical displacement for $p = 0, \tau \neq 0$ although a crack opening does not occur.

For $\varepsilon = 1$ and $\Delta \neq 1$ (different prestressed materials) oscillation occurs, the parameter α characterizing the oscillation zone, here satisfies the inequality $\alpha_*^{-1} < \alpha < \alpha_*$

$$\alpha_* = \max \{f_-/f_+, f_+/f_-\}, \quad f_\pm = \Delta^3 \pm \Delta^2 + \Delta \mp 1$$

The inequality mentioned is obtained under the assumption of the possibility of extreme ratios of the material stiffnesses ($\varepsilon = 0$, $\varepsilon = \infty$). Then $\alpha_* \in (0, \infty)$, if $\Delta > \Delta_* = 0.5437$, where Δ_* is the root of the equation $f_- (\Delta) f_+ (\Delta) = 0$ closest to one.

As $\Delta \to \Delta_*$ the displacements in the body increase without limit, i.e., instability of the piecewise-homogeneous prestressed medium with the crack sets in. Note that the magnitude of the critical strain of a piecewise-homogeneous body Δ_* is identical with the critical strain of a homogeneous Mooney body with a crack /4/. This agreement is due to the fact that the critical strain for a Mooney body is independent of the shear modulus. Computations show that the displacement oscillation zone manifests itself appreciably just for Δ sufficiently close to Δ_*.

Values of the relative oscillation zone width $\Delta x = e^{-2\pi/\ln\alpha}$ /3/ from the crack tip are given in Table 1 for different ε and Δ. As (1.3), (1.6) and (1.8) show, in this case the vertical displacements of the lower and upper crack edges are distinct for $p = 0, \tau \neq 0$. This means that, unlike a homogeneous material, a crack opens under the action of a tangential load.

Table 1

ε	$e = -0.455$	$e = -0.4$	$e = -0.2$	$e = 0.2$
0.5	$1.2\cdot10^{-6}$	$2.0\cdot10^{-8}$	$8.6\cdot10^{-24}$	$3.6\cdot10^{-43}$
0.1	$2.6\cdot10^{-2}$	$3.0\cdot10^{-3}$	$8.3\cdot10^{-10}$	$9.1\cdot10^{-18}$
0.01	$1.9\cdot10^{-1}$	$1.6\cdot10^{-2}$	$3.4\cdot10^{-8}$	$1.2\cdot10^{-15}$

Using (1.4)-(1.9), we obtain for the true additional stresses σ_{SK} /4/

$$\sigma_{yy1} = \theta_{yy1} = \Delta^{-3}M \{2\Delta^3 l_{11} \operatorname{Im} F_5 (z_1) + 2\Delta^2 l_{12} \operatorname{Re} F_6 (z_1) - \Delta (1 + \Delta^4) l_{21} \operatorname{Im} F_5 (z) - (1 + \Delta^4) l_{22} \operatorname{Re} F_6 (z)\} \tag{1.10}$$

$$M^{-1} = \varepsilon (1 + \varepsilon) (\Delta^2 - 1)^2 (\Delta^2 + 1) L (\Delta), \quad z_1 = x + iy\Delta, \quad z = x + iy$$
$$F_5(z) = \left(\frac{z-1}{z+1}\right)^{-1/2} (p \sin \omega - \tau\Delta \cos \omega) + (z^2 - 1)^{-1/2} \times [(2\gamma p + \Delta\tau) \cos \omega - (p - 2\Delta\tau\gamma) \sin \omega]$$
$$F_6(z) = \left(\frac{z-1}{z+1}\right)^{-1/2} (p \cos \omega + \tau\Delta \sin \omega) - p - (z^2 - 1)^{-1/2} [(p - 2\Delta\gamma\tau) \cos \omega + (2\gamma p + \Delta\tau) \sin \omega]$$
$$\omega = \gamma \ln [(z - 1)/(z + 1)]$$
$$\sigma_{yy1} = \theta_{yy1} = \begin{cases} F_6 (x), & |x| > 1 \\ -p, & |x| \leqslant 1 \end{cases}$$

The deduction can be made from (1.10) that the normal stresses have the singularity $\rho^{-1/2} (\rho = \sqrt{(x^2 - 1)^2 + y^2})$ on approaching $x = 1$, $y = 0$. In particular, for $\tau = 0$, the asymptotic σ_{yy1} has the form

$$\theta_{yy1}=\sigma_{yy1}=M\left(M_1\cos\omega+M_2\sin\omega\right)(2\rho)^{-1/2}+O(\rho^{1/2}) \tag{1.11}$$
$$M_1=-4\Delta^3(1+\gamma)\alpha_0^{-1}l_{11}\sin{}^1/_2\varphi_1+2\Delta^2\alpha_0^{-1}l_{12}\cos{}^1/_2\varphi_1+$$
$$2\Delta(1+\gamma)(1+\Delta^4)l_{21}\sin{}^1/_2\varphi-(1+\Delta^4)l_{22}\cos{}^1/_2\varphi$$
$$M_2=2\Delta^3\alpha_0^{+1}l_{11}\sin{}^1/_2\varphi_1-4\gamma\Delta^2\alpha_0^{-1}l_{12}\cos{}^1/_2\varphi_1-\Delta(1+\Delta^4)l_{21}\sin{}^1/_2\varphi+$$
$$2\gamma(1+\Delta^4)l_{22}\cos{}^1/_2\varphi$$
$$x=1+\rho\cos\varphi,\quad y=\rho\sin\varphi,\quad \alpha_0=(\cos^2\varphi+\Delta^4\sin^2\varphi)^{1/4}$$
$$\sin{}^1/_2\varphi_1=\operatorname{sign}\varphi\,[1-\alpha_0^{-2}\cos\varphi]^{1/2}/\sqrt{2}$$
$$\cos{}^1/_2\varphi_1=[1+\alpha_0^{-2}\cos\varphi]^{1/2}/\sqrt{2}$$

The stresses have oscillations near the crack tips on the continuation of the crack line. The other stress components are expressed by the relationships

$$\theta_{yx1}=\Delta^{-2}M\{-\Delta(\Delta^4+1)\,l_{11}\mathrm{Re}F_7(z_1)-(\Delta^4+1)l_{12}\,\mathrm{Im}F_8(z_1)+$$
$$2\Delta l_{21}\,\mathrm{Re}\,F_7(z)+2l_{22}\,\mathrm{Im}F_8(z)\}$$
$$F_7(z)=\left(\frac{z-1}{z+1}\right)^{-1/2}(\rho\sin\omega-\Delta\tau\cos\omega)+\Delta\tau-$$
$$(z^2-1)^{-1/2}[(-2\gamma p+\Delta\tau)\cos\omega+(p+2\gamma\Delta\tau)\sin\omega]$$
$$\theta_{yx1}=\Delta^{-2}M\,\{-2\Delta^5l_{11}\,\mathrm{Re}\,F_7\,(z)-2\Delta^4l_{12}\,\mathrm{Im}\,F_8\,(z_1)+(1+\Delta^4)\cdot$$
$$\cdot\Delta l_{21}\,\mathrm{Re}\,F_7\,(z)+(1+\Delta^4)\,l_{22}\,\mathrm{Im}\,F_8\,(z)]\}$$
$$\theta_{xx1}=\Delta^{-2}M\,[\Phi_1\,(z_1,z)+(\Delta^4+3)\,\Phi_2\,(z_1,z)]$$
$$\Phi_1\,(z_1,z)=2\Delta^3l_{11}\,\mathrm{Im}\,F_5\,(z_1)+2\Delta^2l_{12}\,\mathrm{Re}\,F_6\,(z_1)-\Delta\,(1+$$
$$\Delta^4)\,l_{21}\,\mathrm{Im}\,F_5\,(z)-(1+\Delta^4)\,l_{22}\,\mathrm{Re}\,F_6\,(z)$$
$$\Phi_2\,(z_1,z)=\Delta^3l_{11}\,\mathrm{Im}\,F_9\,(z_1)-\Delta^2l_{12}\,\mathrm{Re}\,F_{10}\,(z_1)-$$
$$\Delta l_{21}\,\mathrm{Im}\,F_9\,(z)+l_{22}\,\mathrm{Re}\,F_{10}\,(z)$$
$$F_9\,(z)=(z^2-1)^{-1/2}\,[(pz+2\gamma\Delta\tau)\sin\omega+(\Delta\tau z+2\gamma p)\cos\omega]-\Delta\tau$$
$$F_{10}\,(z)=(z^2-1)^{-1/2}\,[(pz+2\gamma\Delta\tau)\cos\omega+(2\gamma p-\Delta\tau z)\sin\omega]-p$$
$$F_8(z)=-\left(\frac{z-1}{z+1}\right)^{-1/2}(p\cos\omega+\Delta\tau\sin\omega)+$$
$$(z^2-1)^{-1/2}[(p-2\gamma\tau\Delta)\cos\omega+(2\gamma p+\Delta\tau\sin\omega)]$$
$$\theta_{yx1}=\begin{cases}\Delta^{-1}F_7(x), & |x|>1\\ \tau, & |x|\leqslant 1\end{cases}$$

All the formulas presented for $\varepsilon=1$ (identical materials) reduce to the corresponding formulas in /4/.

2. We will now examine the problem of a crack on the interfacial boundary of two pre-stressed media on the assumption that each medium is described by a model of a compressible semilinear (harmonic) material). In this case the linearized equilibrium equations of the plane problem for a homogeneous initial stress field have the form /5, 7/

$$\frac{\partial\theta_{xx}}{\partial x}+\frac{\partial\theta_{yx}}{\partial y}=0,\quad \frac{\partial\theta_{xy}}{\partial x}+\frac{\partial\theta_{yy}}{\partial y}=0 \tag{2.1}$$

in which

$$\theta_{xx}=\frac{2\mu}{1-2\nu}\left[(1-\nu)\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}\right] \tag{2.2}$$
$$\theta_{yy}=\frac{2\mu}{1-2\nu}\left[\nu\frac{\partial u}{\partial x}+(1-\nu)\frac{\partial v}{\partial y}\right]$$
$$\theta_{xy}=\frac{2\mu}{1+(1-2\nu)\Delta}\left[(1-\nu\Delta)\frac{\partial u}{\partial y}+(1-\nu)\Delta\frac{\partial v}{\partial x}\right]$$
$$\theta_{yx}=\frac{2\mu}{1+(1-2\nu)\Delta}\left[(1-\nu)\Delta\frac{\partial u}{\partial y}+(1-\nu\Delta)\frac{\partial v}{\partial x}\right]$$

Here x, y are Cartesian coordinates in the undeformed body state, u, v are displacement components, μ is the shear modulus, ν is Poisson's ratio, and Δ is the multiplicity of the elongation in the x axis direction for the initial plane state of strain. The boundary conditions of the problem on the interfacial line of the materials $(y=0)$ duplicate (1.1).

It is assumed that the displacements vanish at infinity. As in Sect.1, the problem is reduced by a Fourier transform to the solution of a system of integral equations of the type (1.2), which results, in turn, in the solution of one integral equation

$$2\pi i\psi'(x) = 2\delta \int_{-1}^{1} \frac{\psi(\xi)\,d\xi}{(\xi - x)^2} = -\frac{\pi\Delta a}{m_0\mu_1(z_1 - 1)}(p - i\tau) \tag{2.3}$$

$$\delta = 2\Delta(1 - \nu_2 + \varepsilon(1 - \nu_1)m_0^{-1}, \quad m_0 = \varepsilon(1 - 2\Delta\nu_1) - (1 - 2\Delta\nu_2)$$

$$\psi(x) = v(x) + iu(x), \quad v = v_1|_{y=0} - v_2|_{y=0}, \quad u = u_1|_{y=0} - u_2|_{y=0}$$

Solving the integral equation /6/, we obtain the function $\psi(x)$ whose real and imaginary parts are the desired divergences of the crack edges

$$u = \frac{a\Lambda}{2\mu_1\varepsilon(2\Delta - 1)}(1 - x^2)^{1/2}[p\sin\gamma\omega + \tau\cos\gamma\omega] \tag{2.4}$$

$$v = -\frac{-a\Lambda}{2\mu_1\varepsilon(2\Delta - 1)}(1 - x^2)^{1/2}[p\cos\gamma\omega - \tau\sin\gamma\omega] \tag{2.5}$$

$$\Lambda = \sqrt{m_1 m_2}, \quad \gamma = (2\pi)^{-1}\ln\alpha, \quad \alpha = m_1/m_2 \tag{2.6}$$

$$m_1 = 2\Delta - 1 + \varepsilon(1 + 2\Delta(1 - 2\nu_1)), \quad m_2 = \varepsilon(2\Delta - 1) + 1 + 2\Delta(1 - 2\nu_2), \quad \omega = \ln[(1 + x)/(1 - x)]$$

To analyse the behaviour of the solution near the edges, on the basis of (2.4) and (2.5), we write the displacement formulas for the medium of the lower half-plane

$$u_1 = M\{-al_{11}\operatorname{Im}[pg_1 + \tau g_2] + al_{12}\operatorname{Re}[pg_2 - \tau g_1] - yl_{21}[\operatorname{Re} g_3 + \operatorname{Im} g_4], \quad g_1 = R\sin\omega, \quad g_2 = R\cos\omega - z\} \tag{2.7}$$

$$g_3 = -pr\sin\omega + \tau(r\cos\omega - 1) + (2\gamma p + \tau)R^{-1}\cos\omega + (p - 2\gamma\tau)R^{-1}\sin\omega$$

$$g_4 = -pr\cos\omega + \tau r\sin\omega + (2\gamma p + r)R^{-1}\sin\omega - (2\gamma\tau + p)R^{-1}\cos\omega$$

$$M^{-1} = 2\mu_1(2\Delta - 1)\Lambda\operatorname{ch}\pi\gamma, \quad l_{12} = 2\Delta[(1 - \nu_1)(1 - 2\Delta\nu_2) + (1 - \nu_2)(1 - 2\Delta\nu_1)]$$

$$l_{11} = \varepsilon(2\Delta - 1)x + l_{12}x_1x_2, \quad x_i = 1 + 2\Delta(1 - 2\nu_i)$$

$$l_{21} = x_2 + \varepsilon(2\Delta - 1), \quad \omega = \gamma\ln[(z + 1)/(z - 1)], \quad z = x + iy$$

$$R = (z^2 - 1)^{1/2}, \quad r = ((z + 1)/(z - 1))^{1/2}$$

For $y = 0$ we have from (2.2)

$$u_1 = al_{12}M\{p[(x^2 - 1)^{1/2}\cos\gamma\omega - 1] - \tau(x^2 - 1)^{1/2}\sin\gamma\omega\} \tag{2.8}$$

$$|x| > 1, \quad \omega = \ln[(x + 1)/(x - 1)]$$

$$u_1 = aM\{-l_{11}(1 - x^2)^{1/2}(p\sin\gamma\omega + \tau\cos\gamma\omega)\operatorname{ch}\pi\gamma - l_{12}[p((1 - x^2)^{1/2}\sin\gamma\omega\operatorname{sh}\pi\gamma + x) - \tau(1 - x^2)^{1/2}\cos\gamma\omega\operatorname{sh}\gamma\pi]\} \tag{2.9}$$

$$\omega = \ln[(1 + x)/(1 - x)], \quad |x| \leqslant 1$$

The vertical shifts of the medium are calculated from the formula

$$v_1 = M\{-al_{12}\operatorname{Re}[pg_1 + \tau g_2] + al_{11}\operatorname{Im}[pg_2 - \tau g_1] + yl_{21}[\operatorname{Re} g_7 + \operatorname{Im} g_8]\} \tag{2.10}$$

$$g_7 = p(r\cos\omega - 1) + \tau r\sin\omega + (2\gamma p + \tau)R^{-1}\sin\omega + (p - 2\gamma\tau)R^{-1}\cos\omega$$

$$g_8 = pr\sin\omega - \tau r\cos\omega + (2\gamma p + \tau)R^{-1}\cos\omega - (p - 2\gamma\tau)R^{-1}\sin\omega$$

$$v_1|_{y=0} = -al_{12}M\{p(x^2 - 1)^{1/2}\sin\gamma\omega - q[(x^2 - 1)^{1/2}\cos\gamma\omega - x]\}, \quad |x| > 1 \tag{2.11}$$

$$v_1|_{y=0} = aM\{l_{12}\operatorname{sh}\pi\gamma[p(1 - x^2)^{1/2}\cos\gamma\omega + \tau[(1 - x^2)^{1/2} - x)] + l_{11}\operatorname{ch}\pi\gamma(1 - x^2)^{1/2}(p\cos\gamma\omega - \tau\sin\gamma\omega)\}, \quad |x| \leqslant 1 \tag{2.12}$$

Formulas (2.8), (2.9), (2.10) and (2.12) show that, as in the previous problem, the displacements oscillate as the crack apex is approached. For the analysis of the oscillation zone we note that for $\Delta = 1$ (no prestresses) the oscillation zone parameter α from (2.6) is identical with its value in ordinary elasticity theory /1-3/. For $\varepsilon = 1$ (identical materials) $\alpha = 1$ and there is no oscillation irrespective of the magnitude of the prestress. In the case $\varepsilon = 1$ the coefficient α is within the limits $\alpha_*^{-1} < \alpha < \alpha_*$, where $\alpha_* = (1 + 2\Delta)(1 -$

$2\nu_1)/(2\Delta-1)$. It is seen from (2.7) and (2.10) that for $\Delta = \Delta^* = 0.5$ the plane becomes unstable. On this basis, for $\Delta > \Delta^*$ the parameter α can vary between 0 and ∞, where $\alpha \to \varepsilon(1-\nu_1)/(1-\nu_2)$ as $\Delta \to \Delta^*$.

The dependence of the displacement oscillation zone on the initial strain is given in Table 2 for a semilinear material.

Table 2

ε	$e=-0.49$	$e=-0.4$	$e=-0.2$	$e=0.0$	$e=0.2$
0 5	$8.4\cdot10^{-7}$	$1.1\cdot10^{-8}$	$3.7\cdot10^{-14}$	$7.3\cdot10^{-23}$	$5.8\cdot10^{-39}$
0.1	$2.1\cdot10^{-2}$	$2.3\cdot10^{-3}$	$8.7\cdot10^{-6}$	$2.0\cdot10^{-9}$	$4.8\cdot10^{-16}$
0.01	$1.7\ 10^{-1}$	$1.2\cdot10^{-2}$	$6.6\cdot10^{-5}$	$7.7\cdot10^{-8}$	$2.0\cdot10^{-13}$

Because of their awkwardness, formulas are not presented for the stresses. Analysis shows that the stresses have the classical root singularity at the crack ends and oscillate near them. The oscillation zone width depends on the initial stresses. When the magnitude of the prestress tends to the critical value corresponding to buckling of a body with a crack, the displacement oscillation zone broadens and may cover the whole crack.

REFERENCES

1. WILLIAMS M.L., The stress around a fault or crack in dissimilar media, Bull. Seismol. Soc. America, 49, 2, 1959.
2. ERDOGAN F., Stress distribution in an inhomogeneous elastic plane having a crack, Trans., ASME, Appl Mech., 2, 1963.
3. CHEREPANOV G.P., Brittle Fracture Mechics, Nauka, Moscow, 1974.
4. FILIPPOVA L.M., Stress distribution near crack edges in a prestressed elastic body, PMM, 50, 2, 1986.
5. LUR'YE A.I, Theory of Elasticity, Nauka, Moscow, 1970.
6. MUSKHELISHVILI N.I., Certain Fundamental Problems of Mathematical Elasticity Theory, Nauka, Moscow, 1966.
7. LUR'YE A.I., Non-linear Elasticity Theory. Nauka, Moscow, 1980.

Translated by M.D.F.